Numerical modelling of esker formation in semi-circular subglacial channels

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Framework for sediment transport in R-channels

Assumption:
Water flows through an R-channel fed by a moulin upstream. The bed of the R-channel is a mixed bedrock / alluvial channel and can be fully alluviated.

Numerical model:

Shear stress on the bed (Pa):
\[ \tau_b = f_{bed}v^2 \]
Non-dimensional shear stress:
\[ \tau' = \left( \frac{\rho_s - \rho_w}{\rho_w} \right) \tau \]
If \( \tau'/\tau_c \geq 1 \), motion initiated.

Transport capacity per unit width (m³/s):
\[ q_c = D^{1/2} (\tau' - \tau_c)^{1/2} \]

Volumetric rate sediment transport (m³/s):
\[ q_s = q_c v w \]

Sediment volume per unit length (m³/m):
\[ V_s = \left( V_b + \left( 1 - \lambda \right) n_s \right) W \]

Channel closure by sediment deposition (m³/m²):
\[ v_s = \frac{\partial V_s}{\partial t} \frac{1}{1 - \lambda} \]

System of equations to solve (water conservation, evolution of channel cross-section and sediment conservation):

\[ -\gamma S_h \frac{\partial q_{ch}}{\partial t} = -\frac{\partial Q_{ch}}{\partial x} + \frac{1}{L} \left( \frac{1}{\rho_b} - \frac{1}{\rho_w} \right) - v_n - V_s - \frac{\partial h}{\partial t} \]

\[ \frac{\partial S_h}{\partial t} = v_{max} - v_n - v_s \]

\[ \frac{\partial V_s}{\partial t} = \gamma h \frac{\partial q_{ch}}{\partial x} + \frac{\partial q_{ch}}{\partial x} \]

Hydrology without sediment

\[ Q_{ch} = 50 \text{ m}^3\text{s}^{-1} \]
Constant discharge fed from moulin upstream

Sediment dynamics close to the terminus

Simulation set-up:
- Synthetic melt season
- Constant sediment input upstream (Dsed = 0.17m)
- Wedge-shape glacier

The drop in transport capacity leads to sediment accumulation close to the terminus and the deposition of an incipient esker

Ice geometry and incipient esker deposition

Simulation set-up:
- Same as above with synthetic melt-season
- Simulation run with 4 ice geometries

Results:
- Incipient esker forms for every simulation
- Steeper surface slopes lead to thicker sediment accumulation at 30 km
- Shallower surface slopes lead do deposition occurring further up-glacier

Study summary

Motivation:
- Subglacial water flow deposits, here eskers, can help to understand present subglacial drainage systems

Problem:
- Little is know about sediment transport by subglacial water flow
- A better understanding of these processes would help bridge the gap between eskers and present-day subglacial drainage systems

Goal:
- Develop a numerical framework of sediment transport by water flow in R-channels and explore conditions conducive or detrimental to sediment deposition

Preliminary findings:
- Bottleneck in sediment transport is a natural feature of R-channels
- An incipient esker will form if the sediment supply exceeds the transport capacity at the terminus
- An incipient esker can form at the end of a melt-season
- The ice geometry has a significant influence on the shape of the incipient esker

Ice surface
Bedrock
Ice surface
Shear stress (Pa)
Cross-section (m²)
Distance from water source (km)
Incipient esker
Constant discharge because of thinning ice (less creep closure)

Increase in cross section because of channel size lead to a drop in shear stress and bottleneck in sediment transport

Evolution of incipient esker over time

Constant discharge from water source upstream

Graph: Increase in ice surface due to water flow fed from moulin upstream